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THEORETICAL MODELS OF SURFACE HEAT TREATMENT OF PRODUCTS IN SOLAR FURNACES. 1. UNIFORM RADIANT-ENERGY FLUX

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The problem of determining the rate of heat treatment of a surface, including fusion of it in a solar furnace in the case where a product is immovable and in the case where it moves relative to the focal spot, as a function of the prescribed thickness of the fused layer has been solved by the Goodman heat-balance integral method.

Introduction. One promising trend in industrial utilization of environmentally safe renewable solar energy is the use of solar-energy concentrators (solar furnaces) for realization of technological processes in different temperature ranges: from heat disinfection of sewage at 340–360 K to synthesis of refractory oxides, fullerenes, and other compounds and substances at 3300 K or higher. Investigations in this direction are being conducted in many countries, including the USA, France, Germany, Spain, Israel, Australia, Japan, and a number of CIS countries. The investigations carried out at the Institute of Problems of Materials Science of the National Academy of Sciences of Ukraine have shown that the processes of heat treatment or fusion of materials are promising for formation of coatings for various functional purposes: anticorrosive, wear-resistant, conducting, decorative, and other coatings (see, for example, [1, 2]). In this case, in the surface layers of the products treated, both the material of the product and the substances of which the coating is formed undergo physical and chemical transformations. The quality of the coating formed and of the entire product is dependent on the physicochemical and thermomechanical properties of the main material and the coating substance as well as the characteristics of the heat flux and the regime of its supply: the dimension of the focal spot and the distribution of the energy density in it, instantaneous or gradual establishment of the specified regime of heating, stationary heating of the entire surface or treatment of it by scanning, and so on. An experimental search for an optimum regime of heat treatment is often very difficult and does not always provide a means for solving all problems. A preliminary calculation-theoretical analysis can make the optimization of the process much easier and quicker.

The aim of the present work is to propose relatively simple methods of calculation of the action of heat fluxes on irradiated products which take into account the characteristics of heating in solar furnaces. The first part of this work is devoted to calculating the processes in the case of action of uniform radiant fluxes.

From the standpoint of heat-conduction theory, high-temperature heating of products in solar furnaces is nonlinear even for constant thermophysical properties of the treated materials owing to the substantial reradiation of the heat flux incident on the surface. At the same time, all the best-developed methods of analytical solution of heat-conduction problems are linear. Nonlinear problems are solved as a rule by numerical methods, and generalization of the results of these solutions requires a large number of computer experiments. However, it is always desirable to have the possibility of simple and quick evaluation of the expected results, which cannot be provided by computer experiments. Some nonlinear problems involving radiation from the surface, fusion, or ablation are solved by approximate integral methods [3–8].

Heating of a Stationary Plate. In the case where the surface of a product (it is usually a plate) is located within the boundaries of the focal spot where a radiant heat flux is more or less uniform, we can consider the problem of one-dimensional heating of the plate. For its solution involving allowance for the cooling of the plate by self-radiation and the calculation of the process of fusion once the surface attains the fusion temperature, we use the heat-balance integral method of Goodman [3] as the simplest integral method. In a plane case for constant thermophysical properties we have the heat-conduction equation

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$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2}, \quad a^2 = \frac{\lambda}{\rho c}. \tag{1}$$

The integral methods of solving the nonstationary problems of heat conduction are efficient when the temperature field is conditionally limited by the region with a moving boundary ("heat-penetration depth" according to Biot [8]) beyond which the temperature remains undisturbed (initial), even though this is in contradiction with the diffusion nature of heat conduction. In the present problem, this means that a certain dimension $\delta(t)$, within which the material is warmed up by an external heat flux, is introduced. At $x > \delta$, the temperature remains initial ($T = 0$ if the temperature is measured from the initial level). The integrands methods are applied as a rule to one-dimensional (plane, cylindrical, and spherical) problems. The Goodman method implies that the heat-conduction equation is integrated (formally) in the heated zone, and the integrands are approximated by certain (usually simple) functions satisfying the boundary and other conditions. The unknown approximation parameters are found by solution of the heat-conduction equation written in integral form. Integration of (1) over the thickness of the plate $h = x_2 - x_1$ (here x_2 and x_1, generally speaking, can be dependent on time, for example, in the case of ablation, melting, or solidification) gives

$$\frac{d}{dt} \int_{x_1}^{x_2} T\, dx = T(x_2) \frac{dx_2}{dt} - T(x_1) \frac{dx_1}{dt} + \frac{\lambda}{\rho c} \left[\frac{\partial T}{\partial x}(x_2) - \frac{\partial T}{\partial x}(x_1) \right], \tag{2}$$

where the last term is proportional to the difference between the heat fluxes at the boundaries of the plate determined from the formula

$$q = -\lambda \frac{\partial T}{\partial x}. \tag{3}$$

The approximating temperature profile is taken in the form

$$T = Ta\,[x, a_1(t), a_2(t), a_3(t)]. \tag{4}$$

To determine the form parameters $a_1(t)$, $a_2(t)$ and $a_3(t)$, we have Eq. (2) and two boundary conditions on the surface of the plate. If the number of unknown form parameters is equal, for example, to five, we may require that the initial equation (1) be exactly fulfilled at the boundaries of the plate, i.e., we may substitute the approximating profile into Eq. (1), where we set $x = x_1$ and $x = x_2$.

We set $x_1 = 0$ (the surface of application of the external heat flux) and $x_2 = \delta(t)$ in Eq. 2. In view of the fact that there is no heat flux through the surface $\delta(t)$, we write (2) in the following form:

$$\frac{d}{dt} \int_0^{\delta} T\, dx = \frac{q_0}{\rho c}, \tag{5}$$

where q_0 is the heat flux on the outer surface equal to the difference between the incident flux q_i and the flux of self-radiation from the plate:

$$q_0 = q_{\mathrm{i}} - \varepsilon \sigma T_0^4. \tag{6}$$

Let us assume that the temperature profile has the form

$$T = T_0 F(\xi), \quad \xi = x/\delta, \tag{7}$$

where $\delta(t)$ should be considered as a form parameter. The function $F(\xi)$ satisfies the conditions $F(0) = 1$, $F(1) = 0$, and $F'(1) = 0$, where the prime means differentiation with respect to the argument. As a result, Eq. (5) will be written as follows:

$$\frac{d}{dt}\left[T_0 \delta \int_0^1 F(\xi)\, d\xi\right] = \frac{q_0}{\rho c} . \tag{8}$$

It follows from the determination of the heat flux (3) that

$$q_0 = -\lambda \frac{T_0}{\delta} F'(0) . \tag{9}$$

If the function $F(\xi)$ is prescribed, Eqs. (5), (8), and (9) solve the problem completely since they involve three unknown functions: T_0, δ, and q_0. The success in solving the problem depends on how exactly the form of the function $F(\xi)$ is predicted. Therefore, it is desirable to have a method correcting the form of this function in the process of solution, i.e., to introduce one more form parameter. We take, for example, $F(\xi)$ in the form

$$F(\xi) = (1 - \xi)^n , \tag{10}$$

where n is the time function that is unknown in advance. To determine it, let formula (10) exactly satisfy Eq. (1) at the point $x = 0$, i.e.,

$$\frac{dT_0}{dt} = a^2 \left.\frac{\partial^2 T}{\partial x^2}\right|_0 . \tag{11}$$

As a rule, we obtain an additional equation

$$\frac{dT_0}{dt} = a^2 \frac{T_0}{\delta^2} F''(0) , \tag{12}$$

which allows one to determine the value of n at each instant of time.

System (6), (8), (9), and (12) involves four unknown functions: T_0, δ, n, and q_0, but as long as δ is smaller than the thickness of the plate, it has no length scale and, for a constant external heat flux, time scale, too. Therefore, this problem can be represented in such a form that its solution will be independent of any coefficients. Indeed, let us write these equations in dimensionless form, having introduced the following notation:

$$\tau = \frac{t}{t_*} ; \quad \eta = \frac{\delta}{l_*} ; \quad \theta = \frac{T_0}{T_*} ; \quad T_* = \left(\frac{q_\mathrm{i}}{\varepsilon\sigma}\right)^{1/4} ; \quad l_* = \frac{\lambda T_*}{q_\mathrm{i}} ; \quad t_* = \frac{l_*^2}{a^2} .$$

We note that in this case, the temperature scale is the so-called "radiation-equilibrium temperature" at which the entire heat incident on the surface is reradiated into space. As a result, we obtain

$$\frac{d}{d\tau} \frac{\theta\eta}{n+1} = n \frac{\theta}{\eta} ; \quad \frac{d\theta}{d\tau} = n (n-1) \frac{\theta}{\eta^2} ; \quad n \frac{\theta}{\eta} = 1 - \theta^4 . \tag{13}$$

Combining the first two relations, we can obtain an equation in which there is only the "heat-penetration depth" and the exponent n:

$$\frac{d}{d\tau} \left(\frac{\eta}{n+1}\right)^2 = \frac{4n}{(n+1)^2} . \tag{14}$$

It shows that the "heat-penetration depth" depends on the conditions of heating (temperature, heat flux, and other quantities) only in terms of the exponent n. In the models where n is prescribed, for example, in the work of Biot

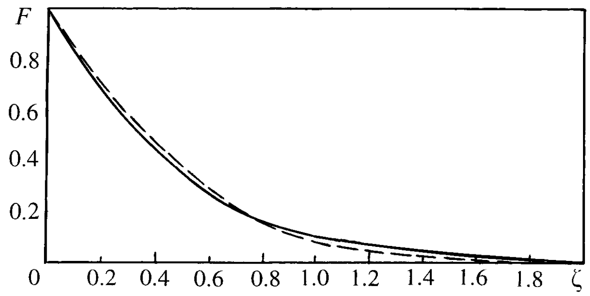

Fig. 1. Exact (solid curve) and approximate (dashed curve) temperature profiles (dimensionless quantities).

[8], $n = 2$, the "heat penetration depth," which quantitatively characterizes the process of heat diffusion, is proportional to $\sqrt{t}$. At the initial stage of heating, when the self-radiation of the surface is negligibly small, system (13) can be solved analytically:

$$\theta = \sqrt{\frac{n+1}{n}\tau}\ ; \quad \eta = \sqrt{n\,(n+1)\,\tau}\ . \tag{15}$$

The exponent n was determined from the condition that the solution is physical, and it was found to be constant and equal to 3. The exact solution of the problem on one-dimensional heating of a semi-infinite body by a constant heat flux has the form

$$F\,(\zeta) = \frac{T}{T_0} = \exp\,(-\zeta^2) - \sqrt{\pi}\,\zeta\,(1 - \mathrm{erf}\,\zeta)\ ; \quad \zeta = \frac{x}{2a\sqrt{t}}; \quad T_0 = \frac{2}{\sqrt{\pi}}\frac{aq_0\sqrt{t}}{\lambda}\ . \tag{16}$$

In the notation introduced, the expression for the surface temperature will be as follows:

$$\theta = \frac{2}{\sqrt{\pi}}\sqrt{\tau}\ . \tag{17}$$

The difference from the approximate solution amounts to 2.3%, which points to a sufficiently good accuracy of this method. Figure 1 shows a comparison of the exact temperature profile (16) with approximation (10), which being expressed in terms of the self-similar variable ζ, has the form

$$F\,(\zeta) = \left(1 - \frac{2\zeta}{\sqrt{n\,(n+1)}}\right)^n .$$

The θ-moment method [5] is somewhat more complex for use, but it is more accurate. For example, in the case of weak influence of the radiation from the heated surface, we obtain the same solution (15), but for the exponent $n = 3.637$. The difference from the exact solution amounts to only 0.066%. At the same time, there is no qualitative difference in behavior between the calculated functions determined by these two approximate methods; therefore, in what follows we will use the simplier method of Goodman. The method proposed can easily be extended to the case of a plate with a finite thickness (see, for example, [6]).

Fusion of the Surface of a Stationary Plate. When the surface of a product is fused, there arises a melt film at the boundary of which with the solid material the following condition of conservation of energy in phase transformation is fulfilled:

$$q_{\mathrm{m}} = q_{\mathrm{w}} + \rho L_{\mathrm{m}} V_{\mathrm{m}}\ , \tag{18}$$

where q_{m} is the heat flux approaching the surface of the body through the melt film and q_{w} is the heat flux going into the solid body.

In the case where the thickness of the film is small, the temperature distribution inside it may be assumed to be linear, i.e., the heat flux entering the film through the outer boundary reaches the boundary with the solid body practically without changes. Consequently,

$$q_{\mathrm{m}} = q_{\mathrm{i}} - \varepsilon\sigma T_0^4$$

and the temperature of the outer surface of the film will be equal to

$$T_0 = T_{\mathrm{m}} + q_{\mathrm{m}}\delta_{\mathrm{m}}/\lambda \,, \tag{19}$$

where δ_{m} is the thickness of the film; $V_{\mathrm{m}} = d\delta_{\mathrm{m}}/dt$.

The boundary between the melt and the solid body, where the temperature is constant and equal to T_{m}, moves with a velocity V_{m}; therefore, the heat-balance equation (5) will take the form

$$\frac{d}{dt}\int_0^{\delta} T\, dx + T_{\mathrm{m}}V_{\mathrm{m}} = -a^2 \left.\frac{\partial T}{\partial x}\right|_0 . \tag{20}$$

The temperature profile in the solid body is approximated in the same manner as before:

$$T = T_{\mathrm{m}}F(\xi)\,, \quad F(\xi) = (1-\xi)^n\,, \quad \xi = \frac{x-\delta_{\mathrm{m}}}{\delta-\delta_{\mathrm{m}}} . \tag{21}$$

It satisfies the following conditions: $T = T_{\mathrm{m}} = \mathrm{const}$ at the boundary with the melt ($x = \delta_{\mathrm{m}}$) and $T = 0$ at the inner boundary of the heated zone ($x = \delta$). To determine $n(t)$ we substitute (21) into Eq. (11), having then set $x = \delta_{\mathrm{m}}$:

$$-V_{\mathrm{m}}F'(0) = \frac{a^2}{\delta-\delta_{\mathrm{m}}}F''(0) . \tag{22}$$

Hence we obtain

$$n = 1 + \frac{V_{\mathrm{m}}(\delta-\delta_{\mathrm{m}})}{a^2} . \tag{23}$$

Transforming Eqs. (18)–(20) and (23) for a constant incident heat flux ($q_i = \mathrm{const}$) with the use of the above-mentioned dimensionless functions, we obtain

$$\frac{d}{d\tau}\left(\frac{\Delta\eta}{n+1}\right) = \frac{1}{\Delta\eta}\,, \tag{24}$$

$$\frac{d\eta_{\mathrm{m}}}{d\tau} = \frac{1}{B_{\mathrm{m}}}\left[1-\theta^4-\frac{n\theta_{\mathrm{m}}}{\Delta\eta}\right], \tag{25}$$

$$\eta_{\mathrm{m}} = \frac{\theta-\theta_{\mathrm{m}}}{1-\theta^4}\,, \tag{26}$$

$$n = 1 + \Delta\eta\,\frac{d\eta_{\mathrm{m}}}{d\tau}\,, \tag{27}$$

TABLE 1. Data on the Time of Heating of a Stationary Plate t_f to the Melting Temperature and the Time of Fusion of a Moving Plate t_m

q_i, MW/m^2	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
t_f, sec	76.2	22.9	11.5	7.08	3.62	2.25	1.55	1.14	0.872
t_m, sec	773	245	128	81.8	45.0	30.1	22.4	17.8	14.7

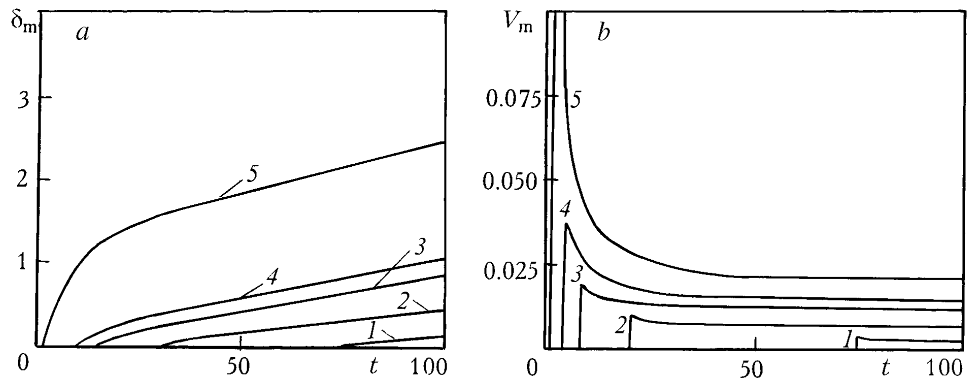

Fig. 2. Time dependence of the thickness of the melt film (a) and the rate of its growth (b) for different heat fluxes: 1) $7\cdot10^5$ W/m^2, 2) $8\cdot10^5$, 3) $9\cdot10^5$, 4) $10\cdot10^5$, and 5) $20\cdot10^5$. δ_m, mm; t, sec; V_m, mm/sec.

where $\eta_m = \delta_m/l_*$, $\theta_m = T_m/T_*$, $\theta = T_0/T_*$, $B_m = L_m/cT_*$, and $\Delta\eta = \eta - \eta_m$.

On passage from the regime of heating to the regime of fusion, the boundary conditions at the solid boundary change abruptly from conditions of the second kind to conditions of the first kind. In this connection, any characteristics of the temperature profile must also change abruptly. Within the framework of the approximation used here, this is the exponent n. At the instant the melting begins ($\tau = \tau_f$), it will be equal to

$$n_f = \frac{B_m + \eta_f (1 - \theta_m^4)}{B_m + \theta_m} .$$

In accordance with this, the rate of fusion at the initial instant will be not zero:

$$\left(\frac{d\eta_m}{d\tau}\right)_f = \frac{1 - \theta_m^4 - \theta_m/\eta_f}{B_m + \theta_m} ,$$

where η_f is the dimension of the heated zone at the instant τ_f.

By way of example, we have calculated the heating and fusion of a ceramic specimen with the following thermophysical properties: ρ = 1500 kg/m^3, λ = 0.6 W/(m·K), ε = 0.8, c = 1200 J/(kg·K), $L_m = 1.5\cdot10^5$ J/kg, and T_m = 1900 K.

For the surface of the product to be able to melt, the radiant heat flux in a solar furnace must exceed a certain minimum level that is determined by the equality between the radiation-equilibrium temperature and the melting temperature:

$$q_{i\,min} = \varepsilon\sigma T_m^4 . \tag{28}$$

For the indicated conditions, $q_{i\,min}$ = 0.59 MW/m^2.

The data on the time of heating to the melting temperature t_m are accumulated in Table 1.

Figure 2 shows the time dependences of the thickness of the melt and the rate of melting for different values of the external heat flux. At the instant the melting begins, the rate of growth of the liquid film is maximum, and then it decreases and becomes practically constant (see curves 5 in Fig. 2).

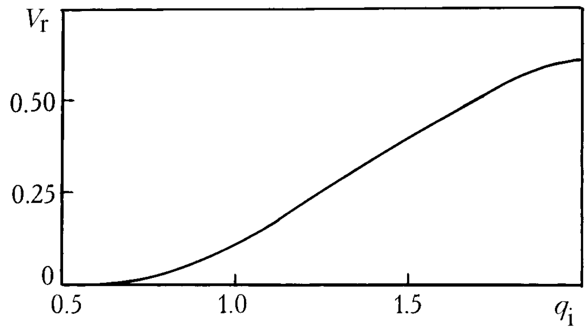

Fig. 3. Dependence of the velocity of travel of the product relative to the focal spot on the heat flux. V_r, mm/sec; q_i, MW/m^2.

Fusion of a Moving Plate. In the case where the area of the focal spot is smaller than the area of the treated surface, it is necessary to move a product (plate) relative to the spot. The problem is to determine the velocity of travel sufficient for the required level of treatment (required thickness of the melt film δ_m). The simplest estimation of this velocity V_r can be made using the results presented in the previous section. Let us assume that the process of heating of each portion of the surface proceeds in the same manner as the process of heating of a stationary plate by a uniform heat flux. In this case, to be melted to a depth δ_m, it must be acted upon by the heat flux for the time t_r equal to the sum of the time of heating to the melting temperature t_m and the time of fusion t_f:

$$t_r = t_f + t_m \,, \tag{29}$$

in this case, t_m is determined from the integral equation since the rate of melting depends on the time:

$$\int_0^{t_m} V_m dt = \delta_m \,.$$

The values of this time characteristic versus the heat flux are presented in Table 1 for δ_m = 1 mm. If the diameter of the spot is equal to d, $V_r = d/(t_f + t_m)$ for points lying on the diameter which is in parallel to the direction of motion. Figure 3 shows the dependence $V_r(q_i)$ for d = 1 cm and δ_m = 1 mm.

It is obvious that at the periphery of the zone acted upon by the focal spot (in the direction perpendicular to the direction of motion) the time of irradiation will be shorter than in the central region. Near the boundary of the spot, the surface regions will not be heated at all to the melting temperature. Therefore, it is desirable to determine the "effective" region of treatment. It can be restricted by the condition that the treated portions of the surface must be acted upon by the heat flux for a time not shorter than the time of heating to the melting temperature. It is easy to determine that the width of the band of such an "effective" treatment h_e will be equal to

$$h_e = d\sqrt{1 - \left(\frac{t_f}{t_f + t_m}\right)^2} \,.$$

In this concrete case, the value of h_e/d is very close to unity because of the large difference between t_f and t_m.

The low rate of treatment of a ceramic plate is due to the fact that most of the heat energy is expended in reradiating (about 90%), and also about 90% of the remaining energy goes to the solid mass, so that less than 1% of all the incident energy is expended directly in melting. Moreover, in the case of radiant heating, the specific expenditure of heat energy is 3 to 4 times lower than in industrial furnaces operated by natural gas [2]. The efficiency of the treatment of a material in solar furnaces at high temperatures (of the order of the radiation-equilibrium temperature) can be substantially increased if the reradiated energy is returned, even if only partially, to the treated surface, especially as the technical solutions of such a problem are, in principle, possible in individual cases.

CONCLUSIONS

1. Supplementing the Goodman heat-balance integral method with the requirement of exact fulfillment of the heat-conduction equation at the characteristic points allows one to solve problems of the dynamics of heating of stationary and moving plane products by a radiant flux in solar furnaces, including the process of fusion of the surface.

2. The problem of determining the velocity of travel of the surface of the treated product relative to the focal spot and the time and other characteristics of the process at a prescribed level of heat treatment (thickness of the fused layer and others) has been solved.

NOTATION

a_i, form parameters of the process; a, thermal-diffusivity coefficient, m^2/sec; c, heat capacity, J/(kg·K); l_*, characteristic length, m; L_m, melting heat, J/kg; t_*, characteristic time, sec; T_*, radiation-equilibrium temperature, K; T_0, temperature of the plate surface, K; T_m, melting temperature, K; V_m, rate of growth of the melt film, mm/sec; q, heat flux, W/m^2; V_r, velocity of travel of the surface, mm/sec; δ, characteristic thickness of the heated layer, m; ε, radiating power; λ, heat-conduction coefficient, W/(m·K); ρ, density, kg/m^3; σ, Boltzmann constant, $W/(m^2 \cdot K)$; η, dimensionless thickness of the heated layer; η_m, dimensionless thickness of the melt film; θ, dimensionless temperature; τ, dimensionless time; ξ, dimensionless coordinate. Subscripts: f, front (of melting); m, melting; r, movement; w, wall; e, effective; i, radiation.

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